THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520G/H University Mathematics 2014-2015
Suggested Solution to Assignment 2

Exercise 10.1

(35) $\displaystyle\lim_{x\to\infty}\frac{x-3}{\sqrt{4x^2+25}}=\lim_{x\to\infty}\frac{1-\frac{3}{x}}{\sqrt{4+\frac{25}{x^2}}}=\frac{1}{2}.$

(36) $\displaystyle\lim_{x\to-\infty}\frac{4-3x^3}{\sqrt{9+x^6}}=\lim_{y\to\infty}\frac{4+3y^3}{\sqrt{9+y^6}}=\lim_{y\to\infty}\frac{\frac{4}{y^3}+3}{\sqrt{\frac{9}{y^6}+1}}=3.$

(83) $\displaystyle\lim_{x\to-\infty}2x+\sqrt{4x^2+3x-2}=\lim_{x\to-\infty}\frac{4x^2-(4x^2+3x-2)}{2x-\sqrt{4x^2+3x-2}}=\lim_{x\to-\infty}\frac{-3x+2}{2x-\sqrt{4x^2+3x-2}}$

$\displaystyle=-\lim_{y\to\infty}\frac{3y+2}{2y+\sqrt{4y^2-3y-2}}=-\lim_{y\to\infty}\frac{3+\frac{2}{y}}{2+\sqrt{4-\frac{3}{y}-\frac{2}{y^2}}}=-\frac{3}{4}$

Exercise 3.6

(37) $\displaystyle\frac{dr}{d\theta}=2\theta\cos\theta^2\cos2\theta-2\sin\theta^2\sin2\theta.$

(45) $\displaystyle\frac{dy}{dt}=10t^9\tan^9t(\tan t+t\sec^2t).$

(55) $\displaystyle\frac{dy}{dt}=6\tan(\sin^3t)\sec^2(\sin^3t)\sin^2t\cos t$

1. (1) When $x>0$, $f(x)=x^2$, $f^{'}(x)=2x$. When $x<0$, $f(x)=-x^2$, $f^{'}(x)=-2x$

   (2)
   $$f^{'}(0)=\lim_{h\to0}\frac{f(h)-f(0)}{h-0}=\lim_{h\to0}|h|=0$$

   (3)
   $$\lim_{x\to0^+}f(x)=\lim_{x\to0^+}2x=0$$
   $$\lim_{x\to0^-}f(x)=\lim_{x\to0^-}-2x=0$$
   $$\lim_{x\to0^+}f(x)=\lim_{x\to0^-}f(x)=f(0)=0$$

   So $f^{'}(x)$ is continuous at $x=0$.

2. Proof: If $x=y$ the inequality holds obviously. Then assume $x>y$, f is continuous in $[y,x]$, and differentiable in (y,x). By the Lagrange mean value theorem, $\exists\xi\in(y,x)$ such that:
   $$f(x)-f(y)=(x-y)f^{'}(\xi)$$
   i.e.
   $$|\sin x-\sin y|=|(x-y)\cos\xi|\le|x-y|$$
   When $y<x$, interchange the role of x and y, can get the same result.

3. Proof:

$$\frac{f(x+h)-f(x)}{h} = \frac{f(x)f(h)-f(x)}{h} = \frac{f(x)(f(h)-1)}{h} = f(x)g(h)$$

$$f^{'}(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} f(x)g(h) = f(x)$$